Optical Phase Estimation using Adaptive Gaussian Measurements to Approach the Quantum Limit

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Optical phase estimation plays a central role in communications [1], sensing [2], and information processing [3]. Photons in squeezed states possess quantum correlations that can be exploited for optical phase estimation to surpass the Standard Noise Limit (SNL), the limit for classical sensors. Furthermore, these quantum states can in principle achieve the ultimate limit in precision allowed by quantum mechanics, known as Heisenberg scaling, which corresponds to the quantum Cramér-Rao bound (QCRB) [4].

Remarkably, homodyne measurements can achieve the QCRB when the phase to be estimated corresponds to a specific, optimal phase, which depends on the squeezing strength of the probe [4]. Previous phase estimation strategies with homodyne detection have leveraged this fact to extend the phase estimation range below the SNL [5, 6]. These strategies use a two-step method, in which the initial step involves a homodyne measurement of N independent squeezed vacuum states $|r,\theta\rangle = \exp(-i\hat{n}\theta)S(r)|0\rangle$, where $\theta \in [0,\pi/2)$ is the optical phase to be estimated, $\hat{n}$ is the photon number operator and $S(r)$ is the squeezing operator [7]. The information obtained from the first step is used to create a rough phase estimate, and then adjust the optical probe closer to the optimal point, thereby allowing for performing more precise subsequent homodyne measurements. However, the performance of these estimation strategies starts to deviate from the QCRB when the phase to be estimated θ differs from the optimal point. Moreover, although the full range of plausible phases that can be encoded in squeezed vacuum states is $[0,\pi)$, homodyne measurements restrict the maximum range of estimation to the interval $[0,\pi/2)$ due to the $\pi/2$ periodicity of the probability distributions of the measurement outcomes.

In this work, we propose a multi-step adaptive Gaussian estimation strategy for optical phase estimation with squeezed vacuum states that approaches the QCRB with a fast convergence rate for any phase encoded in squeezed vacuum. This estimation strategy uses homodyne measurements to implement a comprehensive set of locally optimal POVMs. Then, the strategy performs adaptive optimization based on the Adaptive Quantum State Estimation (AQSE) framework to ensure the asymptotic consistency and efficiency of the estimator for phases $\theta \in [0,\pi/2)$ [8]. Furthermore, by incorporating a localization step using heterodyne measurements, we extend the parametric range to $[0,\pi)$, which is the maximum range of phases that can be encoded in squeezed vacuum, while maintaining an asymptotic quantum optimal performance [9].

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References

[1] S Personick, IEEE Trans. Inform. Theory **17**, 240 (1971)

[2] S Pirandola, B R Bardhan, T Gehring, C Weedbrook and S Lloyd, Nat. Photonics **12**, 724 (2018)

[3] F Chapeau-Blondeau and E Belin, Ann. Telecommun. **75**, 641 (2020)

[4] S Olivares and M G Paris, J. Phys. B **42**, 055506 (2009)

[5] H Nagaoka, in: M Hayashi (ed.), Asymptotic Theory of Quantum Statistical Inference: Selected Papers, World Scientific, 2005, p. 125

[6] A A Berni, T Gehring, B M Nielsen, V Händchen, M G A Paris and U L Andersen, Nat. Photonics **9**, 577 (2015)

[7] D F Walls, Nature **306**, 141 (1983)

[8] A Fujiwara, J. Phys. A **39**, 12489 (2006)

[9] M A Rodríguez-García and F E Becerra, arXiv:2312.07686 (2023)