Quantum State Reconstruction in Discrete and Continuous Spaces: From Ion Traps to Quantum Oscillators

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Quantum state tomography is a fundamental procedure for the verification, validation, and control of quantum devices, forming a critical component in the advancement of quantum computation, simulation, and metrology. However, conventional quantum state estimation techniques, based on full state tomography, suffer from resource requirements that scale exponentially with the system size or the Hilbert space dimension, rendering them infeasible for high-dimensional or highly entangled quantum systems. This motivates the development of scalable and physically informed reconstruction schemes capable of efficiently extracting accurate representations of quantum states using limited measurement data.

In the context of discrete-variable systems, we demonstrate an efficient variational quantum state tomography method based on the matrix product state (MPS) ansatz, tailored for pure-state approximations of one-dimensional many-body systems. We apply this method to reconstruct quantum states generated by a 20-qubit trapped-ion quantum simulator implementing an Ising-type Hamiltonian. Using measurement statistics from only 27 global bases, with 1000 samples per basis, our approach achieves high-fidelity reconstructions while substantially reducing the required measurement and computational overhead (See **??**). The variational optimization within the MPS manifold allows us to accurately capture states with significant entanglement entropy and long-range correlations. Compared to neural network-based quantum state representations—including restricted Boltzmann machines and autoregressive feedforward neural networks—our method exhibits superior convergence behavior, reconstruction accuracy, and robustness to statistical noise.

Simultaneously, we address the limitations of traditional quantum tomography techniques in continuous-variable (CV) systems, such as optical modes described by harmonic oscillators, where standard reconstruction approaches typically project the state onto finite discrete bases and are restricted to low-amplitude regimes due to the exponential growth of the required Hilbert space truncation. To overcome this, we develop a machine-learning-based tomography protocol using feedforward neural networks that directly parameterize the density matrix in the continuous position basis. This enables state reconstruction without discretization, preserving the full infinite-dimensional structure of the CV Hilbert space. A key feature of our approach is the ability to perform localized reconstructions in selected regions of phase space, allowing the allocation of computational resources based on state support. As a result, the scaling of the required measurement and model complexity grows only slowly with the state's energy, allowing access to high-amplitude quantum states that are otherwise intractable using conventional methods.

Together, these two approaches offer complementary strategies for efficient and scalable quantum state tomography in discrete and continuous settings. By leveraging tensor network representations and neural-network-based function approximators, we demonstrate the feasibility of accurate state reconstruction under realistic experimental constraints. Our findings represent a significant step toward enabling real-time, high-fidelity diagnostics of complex quantum systems, particularly in the regimes of non-equilibrium dynamics and quantum many-body quench experiments.