Beyond Mean-Field Effects in the Dynamics of Bose-Einstein Condensates in a Two-Dimensional Double-Core Trap

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We investigate the dynamics of a two-component Bose-Einstein condensate confined in a dual-core, pancake-shaped potential, incorporating beyond-mean-field effects associated with quantum fluctuations. It is assumed that the wavefunction components corresponding to the two species of the binary condensate are identical in each core. Under this assumption, the system is described by a set of linearly coupled two-dimensional Gross-Pitaevskii equations featuring logarithmic nonlinearities, which account for the Lee-Huang-Yang corrections to the mean-field approximation in this setting.

$$\begin{aligned} iu_t + \frac{1}{2}\nabla^2 u - g\ln(|u|^2)|u|^2 u + Kv = 0, \\ iv_t + \frac{1}{2}\nabla^2 v - g\ln(|v|^2)|v|^2 v + Ku = 0, \end{aligned} \tag{1}$$

where K is the tunnel coupling parameter.

For the spatially homogeneous fields $u(t), v(t)$, the processes are described by the two-modes model for the population imbalance $Z = (N_2 - N_1)/N$ and the relative phase of condensates $\theta = \phi_2 - \phi_1$:

$$\begin{aligned} Z_t &= 2K\sqrt{1-Z^2}\sin(\theta), \\ \theta_t &= \frac{gN}{2}[(1-Z)\ln(\frac{N(1-Z)}{2}) - (1+Z)\ln(\frac{N(1+Z)}{2})] - \frac{2KZ}{\sqrt{1-Z^2}}\cos(\theta). \end{aligned} \tag{2}$$

The frequencies of the Josephson oscillations corresponding to the zero- and π-phase modes are determined. Furthermore, the condition for transitioning from Josephson oscillations to the self-trapping (localisation) regime is identified in terms of a critical coupling value Kcr. A detailed phase-plane analysis has been carried out, revealing the existence of distinct dynamical regimes. Bifurcation diagrams and critical points, expressed in terms of the total atom number, have also been obtained. These theoretical predictions are validated through direct numerical simulations. Additionally, a variational approach has been developed to analyse Josephson oscillations between quantum droplets