Measurement Yielding Maximum Information About the State of a System

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We determine the optimal measurement that maximizes the average information gain, also called accessible information, about the state of a qubit system. The qubit is prepared in one of two known states with known prior probabilities. We first develop a rigorous geometric view of the optimization problem which, in turn, provides guidance toward finding an analytical treatment. To treat the problem analytically we employ the formalism developed for the maximum confidence quantum state discrimination strategy and obtain the POVM which optimizes the information gain for the entire parameter space of the system. We show that the optimal measurement coincides exactly with the minimum-error quantum measurement only for two pure states, or when the two states have the same Bloch radius, or they are on the same diagonal of the Bloch disk [1]. As applications, we will discuss the connection to channel capacity [2] and address the question whether quantitative complementarity (generalized wave-particle duality) relations are intrinsic to the quantum state, or measurement related [3].

References

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