Efficient Linear-Optical Generation of Multiqubit GHZ-Like States with Fusion Gates

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Many linear-optical quantum approaches rely on the creation and manipulation of high-dimensional entangled states. Prior research has mainly focused on specific entangled states, such as graph or GHZ states, which form the basis of many notable quantum computing [1] and quantum communication [2] protocols. Although experimentally feasible, existing schemes for generating these states suffer from low success probabilities, resulting in significant technical overhead. This motivates the search for more efficient strategies for preparing entanglement in linear optics.

Recent developments in quantum information theory introduce novel formalisms that represent wider ranges of states and codespaces [3,4], which call for linear optical methods to generate such states. In this paper, we focus on the linear optical generation of multiqubit GHZ-like states, with the traditional maximally entangled GHZ state being a special case:

$$|G_n(\alpha)\rangle = \cos(\alpha)\,|0\rangle^{\otimes n} + \sin(\alpha)\,|1\rangle^{\otimes n}, \quad (1)$$

where $|0\rangle$ and $|1\rangle$ forms a logical basis of the qubit, and the parameter $0 \leq \alpha \leq \pi/4$ is a Schmidt angle that quantifies the degree of entanglement of the state.

These states are useful in algorithms such as quantum teleportation [5], and were recently proven to be local unitary equivalent to weighted hypergraph states [6], a generalization of graph states. Weighted graph states, in turn, have applications in quantum algorithms, including quantum computing [3]. Moreover, these states are covered by the recent extension of Pauli stabilizer formalism [4], a powerful tool lying in the basis of many modern quantum error correction and quantum computing protocols.

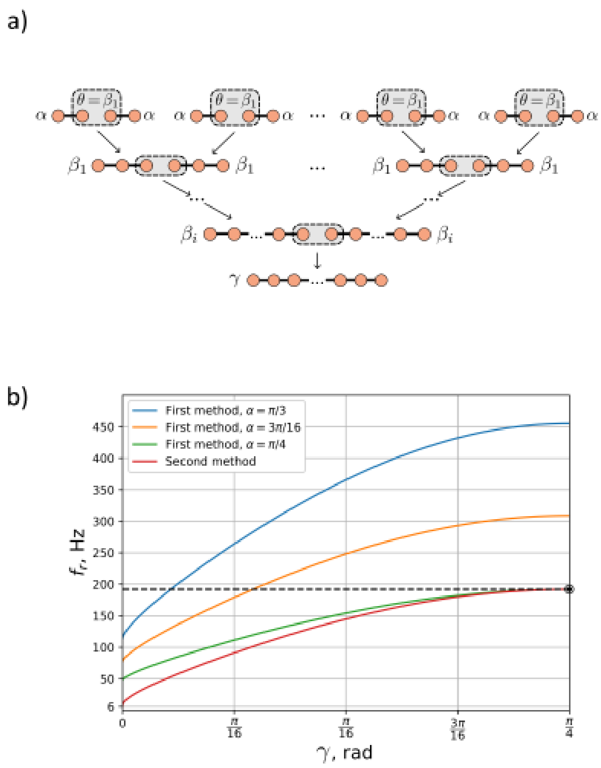

Figure 1: a) First method creating any target GHZ-like state $|G_N(\gamma)\rangle$ from a set of similar arbitrary-entangled 2-qubit resource states $|G_2(\alpha)\rangle$ with fusion operations. Angles in the boxes correspond to the parameters of the modified fusion gates used in the scheme, and boxes without parameters show standard Type-I fusion gates. b) Required generation rate of two-qubit resource states for creating 7-qubit GHZ-like states $|G_7(\gamma)\rangle$ at a target rate of $f_t = 1$ Hz. The dashed line indicates the required Bell pair generation rate to produce maximally entangled states $|G_7(\pi/4)\rangle$ at the same rate using known fusion methods [7]

We propose an approach for creating multiqubit GHZ-like states using well-known linear-optical fusion gates and analyze its performance. Our methods involves sequential application of probabilistic entangling measurements on small pre-generated GHZ-like resource states, fusing them into larger states. We show that:

- The success probabilities of fusion of GHZ-like states may surpass the 50% limit associated with Pauli stabilizer states.
- Our modifications of fusion gates enable control over the entanglement degree in the resulting states.
- Large GHZ-like states can be created with significantly higher efficiency compared to known methods for GHZ state preparation.

We introduce two protocols. The first, depicted in Fig. 1(a), starts with the creation of a number arbitrary-entangled small resource states and applies a sequence of fusion gates on them to construct the target state. The second algorithm uses specific resource states, but achieves higher efficiency. We quantitatively compare both protocols to existing approaches and explore the trade-off between entanglement and success probability.

Fig 1(b) demonstrates required generation rates of two-qubit resource states to achieve a target 7-qubit GHZ-like state generation rate of $f_t = 1$ Hz. Labels show corresponding entanglement degree of resource states for the first method, and an entanglement degree of resource states for the second method equals $\arctan\left(\tan^{1/6}(\gamma)\right)$. In the marginal case $\alpha = \gamma = \pi/4$ both methods correspond to the known schemes creating maximally-entangled GHZ states fusing maximally-entangled resource states [7]. We demonstrate the performance of a such scheme generating 7-qubit GHZ states $|G_7(\pi/4)\rangle$ fusing 6 Bell states with the dashed line in Fig 1(b).

Our estimations show that the efficiency of the state generation can be significantly elevated at the cost of reducing entanglement degree of the target states. These findings suggest that GHZ-like states, with an ability to tune the entanglement degree and improved generation rates, are promising candidates for future linear-optical quantum applications.

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References

[1] S Bartolucci, P Birchall, H Bombin, H Cable, C Dawson, Ms Gimeno-Segovia, E Johnston, K Kieling, N Nickerson, M Pant, F Pastawski, T Rudolph and C Sparrow, Nat. Commun. **14**, 912 (2023)

[2] A Farouk, J Batle, M Elhoseny, M Naseri, M Lone, A Fedorov, M Alkhambashi, S H Ahmed and M Abdel-Aty, Front. Phys. **13**, 130306 (2018)

[3] D Gross, J Eisert, N Schuch and D Perez-Garcia, Phys. Rev. A **76**, 052315 (2007)

[4] M A Webster, B J Brown and S D Bartlett, Quantum **6**, 815 (2022)

[5] A Pathak and A Banerjee, Int. J. Quantum Inf. **9**, 389 (2011)

[6] H Zakaryan, K-R Revis and Z Raissi, arXiv:2408.02740 (2024)

[7] M Gimeno-Segovia, Towards practical linear optical quantum computing, PhD thesis, Imperial College London, 2016